Mathematics exercises (6 PCM + 6 MCB)

1. calculate the first derivative of $y = (\cos x)^x$
2. Find the value of k such that the function defined below is continuous at $x = 0$.

$$f(x) = \begin{cases} \dfrac{1-\cos 2x}{2x^2} & \text{if } x \neq 0 \\ k & \text{if } x = 0 \end{cases}$$

3. Solve for x : $x^4 + x^3 - 4x^2 + x + 1 = 0$
4. Solve in $\mathbb{R}^2$: $\begin{cases} xy = 256 \\ 7(\log_x y + \log_y x) = 50 \end{cases}$

5. Show that $(1+i)^n - (1-i)^n = 2^{\frac{n+2}{2}} \sin n\frac{\pi}{4} i$
6. Find the modulus and argument of:

$$\frac{(\sqrt{3} - i)^3 \cdot (1+i)^2}{1-i}$$

7. Find the inverse M^{-1} if $M = \begin{pmatrix} 1 & 1 & 1 \\ 3 & 4 & -1 \\ 2 & -5 & 3 \end{pmatrix}$

use M^{-1} to solve the system: $\begin{cases} x + y + z = 9 \\ 3x + 4y - z = 13 \\ 2x - 5y + 3z = 8 \end{cases}$

8. Linearise $f(x) = \cos^6 x$
9. For a given set of data: $\Sigma x = 15$, $\Sigma x^2 = 55$, $\Sigma y = 43$, $\Sigma y^2 = 397$, $\Sigma xy = 145$, $n = 5$

Find the equations of the regression lines y on x and x on y.

10. Two ordinary fair dice, on red and one blue, are to be rolled once.

Find the probabilities of the following events:

event A : the number showing on the red die will be a 5 or a 6.

event B : the total of the numbers showing on the two dice will be 7.

11. A box contains six red pens and three blue pens.

(a) A pen is selected at random, the colour is noted and the pen is returned to the box. This procedure is performed a second, then a third time. Find the probability of obtaining

i) three red pens

ii) two red pens and one blue pen, in any order

iii) more than one blue pen.

(b) Repeat (a) but this time find the probabilities if, at each selection, the pen is not returned to the box.

12. Find vector, parametric and cartesian equation of the plane passing through the points $A(1,2,3)$ $B(-2,0,7)$ and $C(3,-4,5)$.

13. Sketch the curve $y = \frac{x^2}{x^2-1}$

14. Solve the equation; $\cos 5x = \sin 4x$

15. Study the convergence or divergence of the series $\sum_{n=1}^{\infty} \frac{1}{(2n-1)(2n+1)}$

END

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Mathematics exercises (6MCB + 6PCM) ①

1. Calculate each of the following limits :

a) $\lim\limits_{x\to+\infty}\left(\dfrac{2+3.4^{x}}{1+4^{x}}\right)$

b) $\lim\limits_{x\to 0}\left(\dfrac{2x+\sin 4x}{x-\sin 2x}\right)$

c) $\lim\limits_{x\to 3}\left(\dfrac{x-3}{\sin \pi x}\right)$

d) $\lim\limits_{x\to-\infty}\dfrac{\sqrt{2+x^{2}}}{2x}$

e) $\lim\limits_{x\to 0^+} x\ln x$

f) $\lim\limits_{x\to \pi/4}(1-\tan x)\sec 2x$

g) $\lim\limits_{x\to 0^+}\dfrac{\ln(\sin x)}{\ln(\tan x)}$

h) $\lim\limits_{x\to 0^+}(x)^{1/\ln x}$

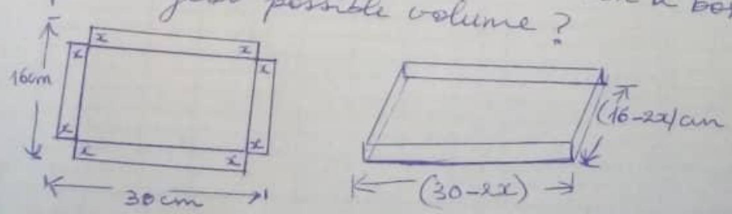
i) $\lim\limits_{x\to+\infty}\dfrac{x\ln x}{x+\ln x}$

j) $\lim\limits_{x\to 0^+}\tan x \ln x$

2. Find the first derivative of the function:

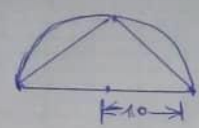
$$f(x) = \left(x^2+3\right)^{1/4}\cdot \tan\left(\frac{x}{2}\right)\cdot\left(\frac{x^3+1}{\sqrt{x}}\right)^{5}\cdot \sin\left(\sqrt{x}\right)$$

3. An open box is to be made from a 16 cm by 30 cm
in piece of cardboard by cutting out squares
of equal size from the four corners and
bending up the sides (figure 1). What size
should the squares be to obtain a box with
largest possible volume ?



(figure 1)

4. A triangle is inscribed in a semicircle of radius 10 so that one side is along the diameter. (figure 2). Find the dimensions of the rectangle with maximum area.



5. Let $z = x+iy$ such that $z^2 + \bar{z}^2 = 4$
Find the equation of the curve satisfying that condition.

6. Find the determinant of the matrix

$$A = \begin{pmatrix} 1 & 1+i & -i \\ 0 & i & 1-2i \\ 1 & 1 & i \end{pmatrix}$$

7. If $[A, B] = AB - BA$. Calculate $[A, B]$ if

$$A = \begin{pmatrix} 1+i & 1 \\ 1 & 1-2i \end{pmatrix} \text{ and } B = \begin{pmatrix} 1 & 0 \\ 2 & -1 \end{pmatrix}$$

8. Find the Values of the real numbers a, b and c so that the function $f(x) = \dfrac{x^2+x-1}{x-1}$ can be expressed as $f(x) = ax+b + \dfrac{c}{x-1}$. Deduce the equations of the ~~eq~~ asymptotes to the curve representing f.

9. Solve in $\mathbb{N}$, the set of natural numbers:
a) $3\,C_{n+1} = 5\,{}_2C_n$ b) $2\,{}_2P_n + 50 = {}_2P_{2n}$