E.SC. MUSANZE

SUBJECT: MATH

CLASS: S5, MCB&PCM GENERAL EXERCISES I

INSTRUCTIONS

- ATTEMPT ALL QUESTIONS IN SECTIONS

1. By expanding $\sin(2x+x)$ and using double angle formula show that $sin3x = 3sinx - 4\sin^3 x$
2. Given the function $f: \mathbb{R} \to \mathbb{R}: x \to \sqrt{(x-1)^2} + \frac{2}{x+1}$
   a) Determine the domain of definition of f
   b) Find the asymptotes on the curve of f.
3. Solve the following system $\begin{cases} xcost - ysint = a \\ xsint + ycost = b \end{cases}$
4. Solve the following trigonometric equation and represent the solution on the trigonometric circle: $\sqrt{3}cosx + 3sinx + 3 = 0$
5. Given that the two real numbers a and b different from zero, let us consider the sequence U_n define by its first term u_0 and the recurrent formula is $u_{n+1} = au_n + b$
   a) Discuss on the formula if a equal to 1
   b) Suppose that a is different to 1, let $k = \frac{b}{1-a}$, and consider the sequence V_n defined by $V_n = u_n - k$, show that the sequence V_n is geometric sequence.
6. In an arithmetic progression the thirteenth term is 27 and the seventh term is three times the second. Find the first term, the common difference and the sum of the first ten terms.
7. Solve in set of real number the equation $2\ln(x+1) = ln(1-x)$
8. The line $x + 2y = 9$ *intersect the curve* $xy + 18 = 0$ at the point A and B.
   Find the coordinates of A and B
9. Solve for a) $4^{5-9x} = \frac{1}{8^{x-2}}$ b) $\log_2(x^2 - 6x) = 3 + \log_2(1-x)$
10. A right triangle ABC is rectangle in A such that AB=22cm and AC=50cm.
    a) find the measure of the angle $A\hat{C}B$
    b) determine the real number n such that BC = $2\sqrt{n}$
11. a) Show that $\sin(x - 60^o) - \cos(30^o - x) = 1$ can be written in the form of $cosx = k$ where k is constant.
    b) hence solve the equation for $0^o < x < 180^o$
12. solve in $\mathbb{R}: 2(lnx)^3 + (lnx)^2 - 5\ln x + 2 = 0$
13. a) Determine whether the series $U_n = \frac{2n+6}{8}$ is arithmetic or geometric
    b) calculate $\sum_{n=1}^{20} U_n$

10. Find the values of x satisfying i) the inequality $2\cos x + 1 > 0$

    ii) the equality $\log_2 \frac{x^2-1}{x+1} = 1$

11. the polynomial $x^4 - 9x^2 - 6x - 1$ *is denoted by* $f(x)$.
    a) Find the value of the constant a for which $f(x) = (x^2 + ax + 1)(x^2 - ax - 1)$

b) Hence the equation f(x)=0, give your answer in exact value.

12. the town of grayrock had population of 10000 in 1960 and 12000 in 1970.

a) assuming an exponential growth model, estimate the population in 1980

b) what is the doubling time for the town's population

13. the low of cooling is $\theta = Ae^{-0.02t}$ *where* $\theta^o c$ *is the excess of temperature* of the water over the temperature of the room temperature at time t minutes and A is constant. Given that the constant room temperature is $20^o c$, and that when t=0 the temperature of the water is $80^0 c$, find the temperature of the water in kelvin when a) t=10, b) t=20, c) t=45

14. the harmonic mean of two numbers is 4, their arithmetic mean is A and geometric mean is G satisfy the relation $2A + G^2 = 27$, find the numbers

15. a) show that $x^3 = 14$ has one root lying between 2 and 3 and can be rearranged in the form $x = \frac{p}{x^2} + \frac{x}{2}$ where p is constant and state the value of p.

b) using the iteration formula $X_{n+1} = \frac{p}{x^2} + \frac{x_n}{2}$ together with your value of p; starting at $x_0 = 2.5$, find to three significant figure a root of $x^3 = 14$

Good Luck!!!

GENERAL EXERCISES II

INSTRUCTIONS

ATTEMPT ALL QUESTIONS IN SECTIONS

Section A

1. Simplify the following: $\frac{\frac{1}{x-4}+\frac{2}{x^2-16}}{3+\frac{1}{x+4}}$
2. Show that A(-6,3), B(3,-5) and C(-1,5) are the vertex of the right angled triangle
3. Solve the equation $\log(x+2) = 1 - \log(x-1)$
4. a) Find A^t *for* $A = \begin{pmatrix} 3 & 2 \\ 4 & -3 \end{pmatrix}$ b) Let $T: V \to W$ be a linear transformation of real vector spaces. Find T(v) and T(w) if T(v+2w)=3v-w and T(v-w)=2v-4w
5. Find the independent term of $(2x^2 - \frac{1}{x})^{12}$

6. Known that $sin\frac{\pi}{4}=\frac{\sqrt{2}}{2}$ $and\ sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$ without using calculator calculate the exact value of $cos\frac{5\pi}{12}$ and $sin\frac{5\pi}{12}$
7. The dimensions of a triangle are 7m, 10m and 16m respectively. Calculate the measures of its angles.
8. In $\mathbb{R}\times\mathbb{R}$, given the system $\begin{cases} kx-9y=-3 \\ 4x+(k-12)y=k \end{cases}$ where k is any real number, which value of k that system has no solution in $\mathbb{R}\times\mathbb{R}$
9. Consider the function $f(x)=\frac{x^2-4x+3}{x-3}$. The function is continue in $\mathbb{R}$? If no redefine the function such that the function continue.
10. Given the function $f:\mathbb{R}\to\mathbb{R}:x\to\log_x\sqrt{1-x^2}$. Determine the value of x where the function is defined.
11. The following are the percentages of 20 samples of milk from different farmers of a certain collection centre of milk:
    4.12 4.04 3.95 3.96 3.98 3.94 3.98 4.12 3.98 3.96
    3.95 4.02 3.95 4.02 4.04 3.98 4.02 4.02 3.95 4.02
    Calculate the standard deviation.
12. Given the curve $y=x+\frac{4}{x}$
    a) Find the asymptotes to the given curve
    b) Find the maximum and the minimum of y
    c) Draw the graph of y
13. Find the real numbers a, b and c of the quadratic function $f(x)=ax^2+bx+c$ whose graph passes through the point (5,0) and has (1,2) as vertex.
14. Find the matrix X such that $2X+3A=B$ if $A=\begin{pmatrix}-2 & -1\\ 1 & 0\\ 3 & -4\end{pmatrix}$ and $B=\begin{pmatrix}0 & 3\\ 2 & 0\\ -4 & -1\end{pmatrix}$

SECTION B

15. Given the numerical function $f(x)=\frac{\sqrt{1-x^2}}{x}$
    a) Determine the domain of definition of f and the limits to the boundaries
    b) Find the equations of asymptotes to the curve.
    c) Determine the first derivative and the second derivative of f.
    d) Determine the variation table of f, the inflexion points and the concavity of the curve represented by the function f.
    e) Draw the graph of the function f.
16. Find the inverse of the following matrix $\begin{pmatrix}1 & 1 & 1\\ 3 & 4 & -1\\ 2 & -5 & 3\end{pmatrix}$
    hence use the inverse found above to solve the following system $\begin{cases} x+y+z=9 \\ 3x+4y-z=13 \\ 2x-5z+3z=8 \end{cases}$
17. Consider the point p(3,4) and circle $x^2+y^2=25$.
    a) Is p a point of the circle?
    b) What is the gradient of the line joining point p and 0(0, 0)?
    c) Find the equation of tangent line to the circle at point p

d) Let Q(x, y) be another point on the circle in the first quadrant, find the slope Mx of the line joining P and Q in terms of x.

e) Calculate the $\lim_{x\to 0} M_x$. how does number relate to your answer?

18. a) Prove that the circle $x^2 + y^2 - 6x - 12y + 40 = 0$ and $x^2 + y^2 - 4y = 16$ are orthogonal.

b) Consider two sequences $\{U_n\}$ and $\{V_n\}$ given by $U_O = 9, U_{n+1} = \frac{1}{2}U_n - 3$ and $V_n = U_n + 6$

c) Show that $\{V_n\}$ is geometric sequence

d)Express $S_n = V_0 + V_1 + V_2 + \cdots + V_n$ in terms of n

GOOD LUCK!!!