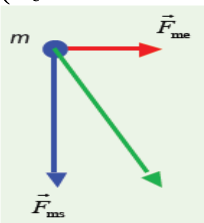
EXERCISES. N°9

SECTION. 1

1. Calculate the effective value of g, the acceleration of gravity,
a)3200m,
b)3200km, above the earth's surface.
2. Determine the net force on the moon ($m_m = 7.36\times10^{22}$ kg) due to the gravitational attraction and both the earth (m_e=5.98×1024 kg) and the sun ($m_s=1.99\times10^{30}$ kg) assuming they are at right angles to each other.



3. What is the effective value of g at a height of 1000km above the earth's surface? That is, what is the acceleration due to gravity of objects allowed to fall freely at this altitude?
4. The universal attraction is given by: $F = G\frac{m_1 m_2}{R^2}$ where $G = 67\times10^{-12}$ USI
a) Find the acceleration g_o at the earth's surface in function of R, M and G, where R is the radius of the earth and M its mass. If R = 6400km, go = 9.81m/s, calculate M.
b) Find in function of g_o, R and h, the acceleration due to the gravity g at a certain height h.
c) If the satellite is at the height h,
(i) Find the speed in function of g_o, R and h.
(ii) Find its value if h = 36000km.
5. Venus is at average distance of 1.08×10^8 km from the sun. Estimate the length of the Venusian year using the fact that the earth is 1.49×10^8 km.
6. The planet Mars of mass m describes around the sun of mass M, an ellipse of mean radius of orbit $a = 230\times10^6$ km in 1.8 years. The satellite Deimos of mass m' describes around the planet mars an ellipse of mean radius $a' = 28\times10^6$ km in 30h. Find the mass of the planet mars, given that $M = 2\times10^{30}$ km and 1year is 365days.
7. We actually know fifteen satellites revolving around the planet Uranus. Let us denote the period of revolution of satellite by T and the mean distance to the centre of the planet by r. The five bigger than others have the following characteristics:

Satellite	Oberon	Titania	Umbriel	Ariel	Miranda
T (J)	13.46	8.706	4.144	2.520	1.414
r (10^3 km)	582.6	435.8	266.0	191.2	129.8

a) (i) For each satellite, calculate T_2 and r_3,
ii) Assume $T_2 = y$ and r_3 = x. Trace the graph of y = f(x). What conclusion related to the nature of the graph can you get?
b) i) Calculate the slope of the plotted segment.
ii) Deduce the mass of Uranus.
8.Use Kepler's second law to convince yourself that the Earth must move faster in its orbit during December, when it is closest to the Sun, than during June, when it is farthest from the Sun.
9.The gravitational force that the Sun exerts on the Moon is about twice as great as the gravitational force that the Earth exerts on the Moon. Why doesn't the Sun pull the Moon away from the Earth during a total eclipse of the Sun?
10.A satellite in orbit is not truly traveling through a vacuum. It is moving through very, very thin air. Does the resulting air friction cause the satellite to slow down?
11.At what position in its elliptical orbit is the speed of a planet a maximum? At what position is the speed minimum?
12. If you are given the mass and radius of planet X, how would you calculate the free-fall acceleration on the surface of this planet?

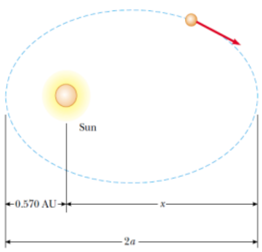
SECTION 2. Newton's Law of Universal Gravitation

1. Determine the order of magnitude of the gravitational force that you exert on another person 2 m away. In your solution state the quantities you measure or estimate and their values.
2. Two ocean liners, each with a mass of 40 000 metric tons, are moving on parallel courses, 100 m apart. What is the magnitude of the acceleration of one of the liners toward the other due to their mutual gravitational attraction? Treat the ships as particles.
3. A 200kg object and a 500kg object are separated by 0.4m.
a) Find the net gravitational force exerted by these objects on a 50kg object placed midway between them.
b) At what position (other than an infinitely remote one) can the 50kg object be placed so as to experience a net force of zero?

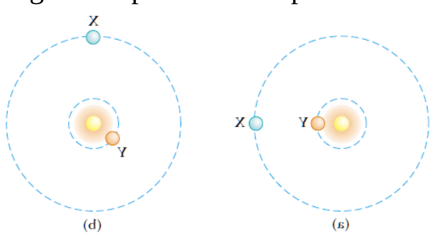
4. Two objects attract each other with a gravitational force of magnitude 1×10^{-8} N when separated by 20cm. If the total mass of the two objects is 5kg, what is the mass of each?
5. During a solar eclipse, the Moon, Earth, and Sun all lie on the same line, with the Moon between the Earth and the Sun.
a) What force is exerted by the Sun on the Moon?
b) What force is exerted by the Earth on the Moon?
c) What force is exerted by the Sun on the Earth?

SECTION 3. Kepler's Laws and the Motion of Planets

1. The center-to-center distance between Earth and Moon is 384 400 km. The Moon completes an orbit in 27.3 days.
a) Determine the Moon's orbital speed.
b) If gravity were switched off, the Moon would move along a straight line tangent to its orbit, as described by Newton's first law. In its actual orbit in 1s, how far does the Moon fall below the tangent line and toward the Earth?
15. Io, a moon of Jupiter, has an orbital period of 1.77 days and an orbital radius of 4.22×10^5 km. From these data, determine the mass of Jupiter.
2. Comet Halley in figure approaches the Sun to within 0.570 AU, and its orbital period is 75.6 years. (AU is the symbol for astronomical unit, where 1 AU=1.5×10^{11} m is the mean Earth–Sun distance.) How far from the Sun will Halley's comet travel before it starts its return journey?



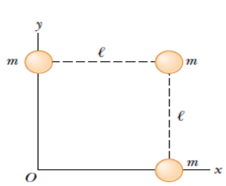
3. Two planets X and Y travel counterclockwise in circular orbits about a star as in Figure. The radii of their orbits are in the ratio 3:1. At some time, they are aligned as in Figure (a), making a straight line with the star. During the next five years, the angular displacement of planet X is 90°, as in Figure (8b). Where is planet Y at this time?



4. Neutron stars are extremely dense objects that are formed from the remnants of supernova explosions. Many rotate very rapidly. Suppose that the mass of a certain spherical neutron star is twice the mass of the Sun and its radius is 10.0 km. Determine the greatest possible angular speed it can have so that the matter at the surface of the star on its equator is just held in orbit by the gravitational force.
5. As thermonuclear fusion proceeds in its core, the Sun loses mass at a rate of 3.64×10^9 kg/s. During the 5000yr period of recorded history, by how much has the length of the year changed due to the loss of mass from the Sun? *Suggestions:* Assume the Earth's orbit is circular. No external torque acts on the Earth–Sun system, so its angular momentum is conserved. If x is small compared to 1, then $(1+x)^n$ is nearly equal to $1+ nx$.

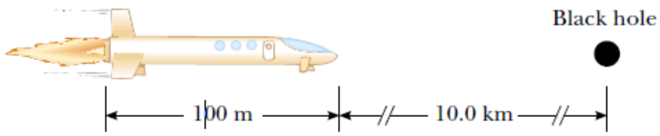
SECTION .4 The Gravitational Field

1. Three objects of equal mass are located at three corners of a square of edge length L as in Figure. Find the gravitational field at the fourth corner due to these objects.



2. A spacecraft in the shape of a long cylinder has a length of 100 m, and its mass with occupants is 1 000 kg. It has strayed too close to a black hole having a mass 100 times that of the Sun (Fig. P13.24). The nose of the spacecraft points toward the black hole, and the distance between the nose and the center of the black hole is 10.0 km.
a) Determine the total force on the spacecraft.

b) What is the difference in the gravitational fields acting on the occupants in the nose of the ship and on those in the rear of the ship, farthest from the black hole? This difference in accelerations grows rapidly as the ship approaches the black hole. It puts the body of the ship under extreme tension and eventually tears it apart.

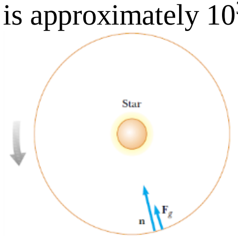


3. A satellite of the Earth has a mass of 100 kg and is at an altitude of 2×10^6 m.
a) What is the potential energy of the Satellite-Earth system?
b) What is the magnitude of the gravitational force exerted by the Earth on the satellite?
c) What If? What force does the satellite exert on the Earth?
4. How much energy is required to move a 1000kg object from the Earth's surface to an altitude twice the Earth's radius?
5. After our Sun exhausts its nuclear fuel, its ultimate fate may be to collapse to a *white dwarf* state, in which it has approximately the same mass as it has now, but a radius equal to the radius of the Earth. Calculate:
a) the average density of the white dwarf,
b) the free-fall acceleration,
c) the gravitational potential energy of a 1kg object at its surface.
6. How much work is done by the Moon's gravitational field as a 1000kg meteor comes in from outer space and impacts on the Moon's surface?

SECTION .5 Energy Considerations in Planetary and Satellite Motion

1. A 500-kg satellite is in a circular orbit at an altitude of 500 km above the Earth's surface. Because of air friction, the satellite eventually falls to the Earth's surface, where it hits the ground with a speed of 2.00 km/s. How much energy was transformed into internal energy by means of friction?
2. A satellite of mass *m*, originally on the surface of the Earth, is placed into Earth orbit at an altitude *h*.
a) With a circular orbit, how long does the satellite take to complete one orbit?
b) What is the satellite's speed?
c) What is the minimum energy input necessary to place this satellite in orbit? Ignore air resistance but include the effect of the planet's daily rotation. At what location on the Earth's surface and in what direction should the satellite be launched to minimize the required energy investment? Represent the mass and radius of the Earth as M_E and R_E.
3. A 1 000-kg satellite orbits the Earth at a constant altitude of 100 km. How much energy must be added to the system to move the satellite into a circular orbit with altitude 200 km?
4. Determine the escape speed for a rocket on the far side of Ganymede, the largest of Jupiter's moons (Figure P13.41). The radius of Ganymede is 2.6×10^6 m, and its mass is 1.495×10^{23} kg. The mass of Jupiter is 1.9×10^{27} kg, and the distance between Jupiter and Ganymede is 1.071×10^9 m. Be sure to include the gravitational effect due to Jupiter, but you may ignore the motion of Jupiter and Ganymede as they revolve about their center of mass.

5. In Larry Niven's science-fiction novel *Ringworld,* a rigid ring of material rotates about a star (Fig). The tangential speed of the ring is 1.25×10^6 m/s, and its radius is 1.53×10^{11} m.
a) Show that the centripetal acceleration of the inhabitants is 10.2 m/s2.
b) The inhabitants of this ring world live on the starlit inner surface of the ring. Each person experiences a normal contact force **n**. Acting alone, this normal force would produce an inward acceleration of 9.9m/s^2. Additionally, the star at the center of the ring exerts a gravitational force on the ring and its inhabitants. The difference between the total acceleration and the acceleration provided by the normal force is due to the gravitational attraction of the central star. Show that the mass of the star is approximately 10^{32} kg.



6. As an astronaut, you observe a small planet to be spherical. After landing on the planet, you set off, walking always straight ahead, and find yourself returning to your spacecraft from the opposite side after completing a lap of 25km. You hold a hammer and a falcon feather at a height of 1.4m, release them, and observe that they fall together to the surface in 29.2s. Determine the mass of the planet.

7. Many people assume that air resistance acting on a moving object will always make the object slow down. It can actually be responsible for making the object speed up. Consider a 100kg Earth satellite in a circular orbit at an altitude of 200 km. A small force of air resistance makes the satellite drop into a circular orbit with an altitude of 100 km.

a) Calculate its initial speed.

b) Calculate its final speed in this process.

c) Calculate the initial energy of the satellite–Earth system.

d) Calculate the final energy of the system.

e) Show that the system has lost mechanical energy and find the amount of the loss due to friction.

f) What force makes the satellite's speed increase? You will find a free-body diagram useful in explaining your answer.

8.a) Determine the amount of work (in joules) that must be done on a 100kg payload to elevate it to a height of 1000 km above the Earth's surface.

b) Determine the amount of additional work that is required to put the payload into circular orbit at this elevation.

9. X-ray pulses from Cygnus X-1, a celestial x-ray source, have been recorded during high-altitude rocket flights. The signals can be interpreted as originating when a blob of ionized matter orbits a black hole with a period of 5ms. If the blob were in a circular orbit about a black hole whose mass is $20M_{Sun}$, what is the orbit radius?

10. A spherical planet has uniform density. Show that the minimum period for a satellite in orbit around it is $T_{min} = \sqrt{\frac{3\pi}{G\rho}}$ independent of the radius of the planet.

11.Two stars of masses M and m, separated by $T^2 = \frac{4\pi^2 d^3}{G(M+m)}$ a distance d, revolve in circular orbits about their center of mass (Fig). Show that each star has a period given by Proceed as follows: Apply Newton's second law to each star. Note that the center-of-mass condition requires that $Mr_2 = mr_1$, where $r_1 + r_2 = d$.