

Mathematics S4 PCM & MC B

Unit 1. Fundamentals of trigonometry

1) Convert the following degree measures to radians in terms of π

- a) 3° b) 225° c) 130° d) 36°

2) Convert the following radian measures to degrees

- a) $\frac{\pi}{5}$ b) $\frac{3\pi}{4}$ c) $\frac{\pi}{18}$ d) $\frac{5\pi}{6}$

3) Determine if possible, the trigonometric ratios of

- a) 0° b) 90° c) 45° d) 120°

4) Given a) $\sin \theta = 0.8$ find the possible values of $\cos \theta$ and $\tan \theta$

- b) $\cos \theta = 0.5$ " " " $\sin \theta$ "
 c) $\tan \theta = -2$ " " " $\sin \theta$ and $\cos \theta$

5) Simplify the expressions

- a) $\cos \theta \tan \theta$ b) $\frac{\sin \theta}{\tan \theta}$ c) $\frac{\cos \theta \tan \theta + \sin \theta}{\tan \theta}$

6) Find in each of the following, four possible values of θ for which

- a) $\sin \theta = 0$ b) $\cos \theta = 0$ or $\sin \theta = 0$ c) $\tan \theta$ does not exist

7) Simplify the following

- a) $\sin^2 2A + \cos^2 2A$ b) $\cos^2 \theta + 1$ c) $\frac{\sin \theta}{\csc \theta} + \frac{\cos \theta}{\sec \theta}$ d) $(\sin A + \cos A)^2 + (\sin A - \cos A)^2$

8) Find in simplest surd form, the value of each of the following

- a) $\cos 105^\circ$ b) $\cos 75^\circ$ c) $\sin 15^\circ$ d) $\sin 75^\circ$

9) If α is an angle in quadrant 1 and β is an angle in quadrant 2 such that $\cos \alpha = \frac{3}{5}$ and $\sin \beta = \frac{3}{5}$, evaluate each of the following without the use of a calculator

- a) $\cos(\alpha + \beta)$ b) $\sin(\alpha - \beta)$ c) $\cos(\alpha - \beta)$

10) Simplify $\sin \theta \cos 2\theta + \cos \theta \sin 2\theta$

- b) $\cos \alpha \cos(90^\circ - \alpha) - \sin \alpha \sin(90^\circ - \alpha)$

11) Angle A is an acute angle and $\sin A = \frac{7}{25}$, Angle B is obtuse and $\sin B = \frac{4}{5}$ find an exact expression for

- a) $\sin(A+B)$ b) $\cos(A+B)$ c) $\tan(A+B)$

12) solve the triangles with the following measures

a) $a = 5\text{cm}$ $b = 6\text{cm}$ $C = 45^\circ$

b) $a = 12\text{cm}$ $B = 57^\circ$ $c = 15\text{cm}$

c) $A = 52^\circ$ $a = 6\text{cm}$ $B = 67^\circ$

d) $C = 71^\circ$ $c = 15\text{cm}$ $b = 5\text{cm}$

13) a) From a tower of 32m of height, a car is observed at an angle of depression of 55° . Find how far the car is from the tower.

b) A town B is 13km south and 12km west of a town A. Find the bearing and distance of B from A.

c) Mutesi is standing on the bank of a river and observes that the angle subtended by a tree on the opposite bank is 60° . When she retreats 40m from the bank, she finds the angle to be 30° . Find the height of the tree and the breadth of the river.

Unit 2 Propositional and predicate logic

- 1) Which of the following are propositions. Give reasons for your answers.
a) $27+35=62$ b) The population of Rwanda is less than 2 millions
c) Is Jupiter round? d) 6 is greater than 16 e) m is greater than n
- 2) Construct the truth tables for each of the following statements:
a) $p \wedge (vq)$ b) $\neg(p \wedge (wq))$ c) $p \wedge (qv \wedge r)$ d) $\neg[(\neg p) \vee (\neg q)]$
e) $(p \Rightarrow q) \wedge (q \Rightarrow r)$
- 3) Show that the following are tautologies
a) $p \vee (\neg p)$ b) $(p \vee q) \vee [(w \wedge p) \wedge (w \wedge q)]$
- 4) Represent the truth of each of the following statements by means of a Venn diagram.
i) All human beings are mortal and x is a human being
ii) No policeman is a thief
iii) All natural numbers are rational numbers
- b) Use the Venn diagram to examine the validity of the argument
 S_1 : All professors are absent-minded
 S_2 : Mutesi is not a professor
 S : Mutesi is absent-minded
5. Construct the circuit for the statements
a) $p \vee (q \vee r)$ b) $p \wedge (q \vee r)$

Unit 3. Binary operations.

- 1) Determine whether the following are groups or not
a) odd integers under addition
b) the set of integers under multiplication
c) the set of rational numbers under addition.
- 2) Using a Cayley table displaying different composition of the following functions $f(x)=x$, $g(x)=-x$ and $h(x)=\frac{1}{x}$ of the set $S=\{f, g, h\}$ determine whether the composition on the set S is
a) closure b) associative c) admits the identity element (and find it)
d) admits the inverse element e) commutative.

Unit 4. Set \mathbb{R} of real numbers

1) Solve the following.

a) $|5-2| = \frac{x}{2}$

b) $|3-2x| = 5$

c) $|2x+7| \geq 3$

d) $|9-3x| \geq 0$

e) $|4x-1| < 15$

f) $|2x+3| \geq |x-4|$

2) Simplify and give the answer in rational form

a) $\frac{\sqrt{5}}{\sqrt{10}}$

b) $\frac{2}{2\sqrt{3}-3}$

c) $\frac{1}{\sqrt{7}-\sqrt{3}}$

d) $\frac{4\sqrt{3}}{2\sqrt{3}-\sqrt{3}}$

e) $\frac{1}{3\sqrt{2}-2\sqrt{3}}$

f) $\frac{\sqrt{3}}{\sqrt{2}(\sqrt{6}-\sqrt{3})}$

3) Without using a calculator, find:

a) $\log 0.001$

b) $\log\left(\frac{1}{\sqrt{10}}\right)$

c) $\log(1000 \sqrt[3]{10})$

d) $\log\left(\frac{10^2}{10^6}\right)$

4) Express each of the following as a single logarithm

a) $\log 8 + \log 6 - \log 3$

b) $\log 56 - \log 7 - \log 4$

5) a) The population of a town in 2013 was estimated to be 35000 people. It has an annual rate of increase (growth) of about 2.4%.

a) What is the growth factor for the town?

b) Write an equation to represent future growth.

c) Use your equation to estimate the population in 2027 to the nearest hundred people.

b) Nurserwa bought a new car at a cost of 17 850 000 Fmw. The car depreciates by approximately 15% of its value each year.

a) What is the depreciation factor for the value of this car?

b) Write an equation to model the depreciation value of this car.

c) What will the car be worth in 10 years?

c) Jose invests 500 000 Fmw at a bank offering 10% compounded quarterly. Find the amount of the investment at the end of 5 years (if untouched).

Unit 5: Linear equations and inequalities

1. Solve the following equations and inequalities

a) $x^2 - 5x = 0$

b) $\frac{3-x}{2x-7} = 0$

c) $2x + 6 \geq x - 5$

d) $\frac{6+4x}{12} \geq \frac{3+4x}{-12}$

2) Solve and discuss

a) $(2-3m)x + 1 = m^2(1-x)$

b) $(m+3)x \geq 2$

3) Solve the following simultaneous equations

a) $\begin{cases} x+y=8 \\ x-y=2 \end{cases}$

b) $\begin{cases} y+1=2x \\ x-y+3=0 \end{cases}$

c) $\begin{cases} 5x+3y=12 \\ 2y+7x=19 \end{cases}$

4) A construction company transports material to different sites. The cost of transporting sand to site 1 is 6500 FRW per trip; while gravel is transported to the same site at 10000 FRW per trip. The cost of transporting sand to site 2 is 10000 FRW per trip while gravel is transported at 13000 FRW per trip. Assuming the total costs to site 1 and 2 were 63000 FRW and 90000 FRW respectively, how many trips did the vehicle make to each site?

5) The coca-cola bottling company distributes x crates of Fanta and y crates of coke within Kigali area every day. The daily distribution of Fanta is 100 crates less than of coke. If the company distributes 400 crates daily, how many crates can be distributed for each brand?